Lecture 1

Dynamics

Kinematics

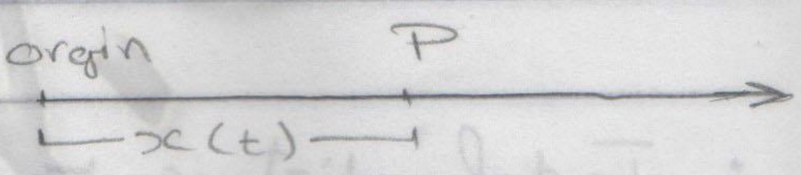
Study The Properties of motion regardless its reasons (velocity, acceleration...)

Kinetics

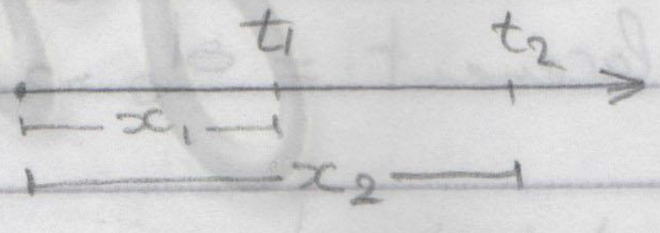
Relates The Properties of motion with its reasons (Newton's Second Law)

Kinematics of a Particle

* **Rectilinear motion:** "motion in St. Line"



1-) Position $x(t)$



2-) Displacement: $d = \Delta x = x_2 - x_1$

3-) Velocity $v(t) = \frac{dx}{dt}$

4-) Acceleration $a(t) = \frac{dv}{dt}$

Ex.: A Particle moves in a St. Line such that $x(t) = t^2 - 2t + 5$

calculate: 1-) initial position, Velocity & acceleration

2-) The time when velocity is zero, Position & acceleration at $v = 0$

3-) The position, displacement & Total distance taveled at $t = 3$ s

Solution

① $x(t) = t^2 - 2t + 5$ m

$v(t) = \frac{dx}{dt} = 2t - 2$ m/s

$a(t) = \frac{dv}{dt} = 2$ m/s²

at $t = 0$ $x(0) = 5$ m , $v(0) = -2$ m/s

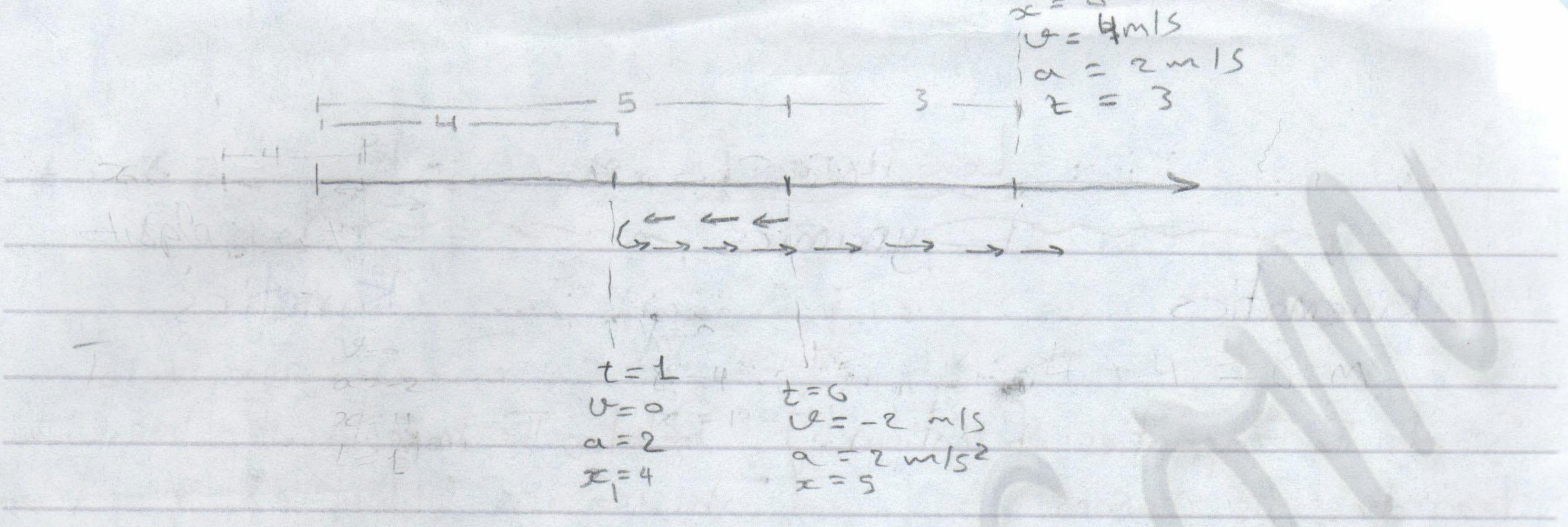
$a(0) = 2$ m/s²

② at $v = 0$

$v(t) = 2t - 2 = 0 \Rightarrow t = 1$ sec.

at $t = 1$ s , $a = 2$ m/s² , $x = 1 - 2 + 5 = 4$ m

$v = 0$ m/s



at t=3 $x = 3^2 - 2(3) + 5 = 8$ m

$v = 2(3) - 2 = 4$ m/s

$a = 2$ m/s²

∴ Total distance $= \overleftarrow{1} + \overrightarrow{4} = 5$ m

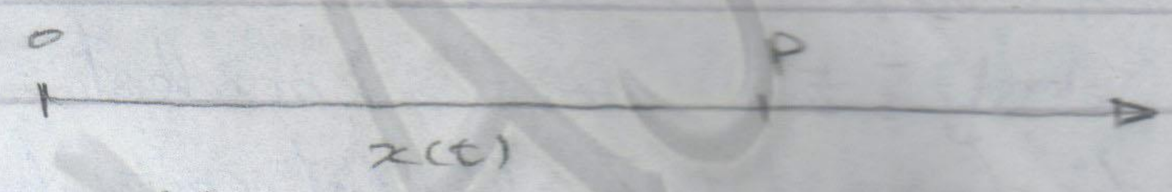
displacement $= d_{0\to 3} = x_3 - x_0 = 8 - 5 = 3$ m

Total dist. (or) $= |x_{v=0} - x_{initial}| + |x_{Final} - x_{v=0}|$

$= |x_1 - x_0| + |x_3 - x_1| = |4 - 5| + |8 - 4|$

$= 1 + 4 = 5$

Summary — Tutorial



$x(t) \xrightarrow{\frac{dx}{dt}} v(t) = \frac{dx}{dt} \xrightarrow{\frac{d}{dt}} a(t) = \frac{dv}{dt}$

$\xleftarrow{\int dt}$ $\xleftarrow{\int dt}$

→ Displacement : $= x_{Final} - x_{initial}$

→ at $v=0$ ⟶ direction must be changed

Problem (11-14) P.614 : A Particle moves in St.Line $a = 9 - 3t^2$

it starts at $t=0 \longrightarrow v_0 = 0$, $x_0 = 5$ determine

① Time when velocity is again Zero

② Position & velocity at $t = 4s$

③ displacement & Total distance Travelled at $t = 4s$

Soln.

$$\frac{dv}{dt} = \boxed{a = 9 - 3t^2}$$

$$\int dv = \int (9 - 3t^2)\, dt$$

$$v = 9t - t^3 + C \quad , \because \text{at } v_0 = 0$$

$$0 = 0 - 0 + C \Rightarrow C = 0$$

$$\therefore \boxed{v(t) = 9t - t^3}$$

$$\frac{dx}{dt} = v = 9t - t^3$$

$$\int dx = \int (9t - t^3)\, dt$$

$$x = 9/2\, t^2 - 1/4\, t^4 + C \quad , \because \text{at } x_0 = 5$$

$$5 = 0 - 0 + C \Rightarrow C = 5$$

$$\therefore \boxed{x = 4.5\, t^2 - 0.25\, t^4 + 5}$$

$\therefore$ ① at $t = 0$

$$9t - t^3 = 0 \Rightarrow t(9 - t^2) = 0$$

$$\boxed{t = 0} \text{ (crossed out)}, \quad \boxed{t = 3}, \quad \boxed{t = -3} \text{ (crossed out)}$$

$\therefore$ at $\boxed{t = 3}$ it velocity equal Zero

$$x(3) = 4.5(3)^2 - 0.25(3)^4 + 5 = 25.25 \text{ m}$$

$$a_3(3) = \text{Zero} \quad , v_3 = 0$$

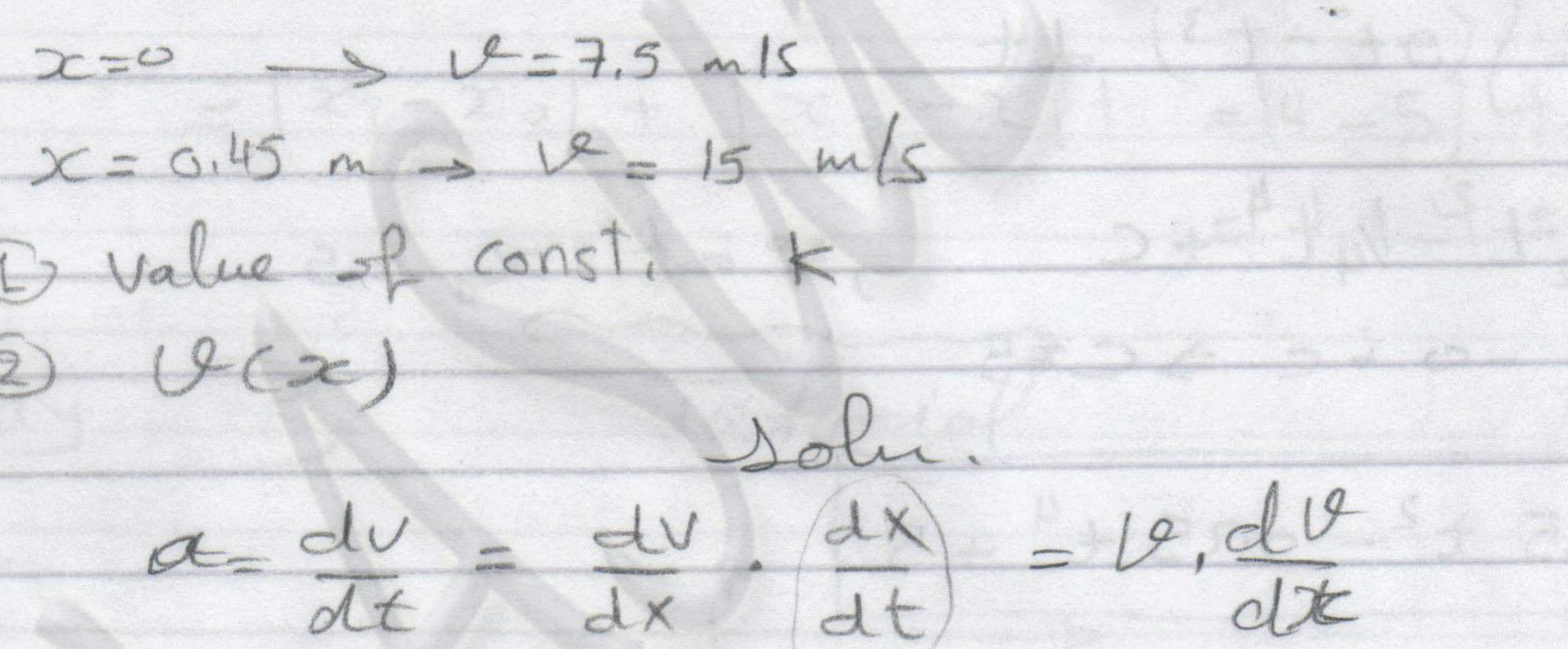
② at $t = 4s$

$$x_4 = 13 \text{ m} \qquad a_4 = -3 \text{ m/s}^2$$

$$v_4 = -28 \text{ m/s}$$

$$\text{displacement} = x_{Final} - x_{Init.} = x_4 - x_0 = 13 - 5 = 8 \text{ m}$$

$$\text{Total dist.} = |x_4 - x_3| + |x_3 - x_0|$$
$$= |13 - 25.25| + |25.25 - 5|$$
$$= 32.5 \text{ m}$$



Prob. (11.18) P. 614 : The Acceleration at st. line $a = 600x + 600kx^3$ K is const.

at $x = 0 \rightarrow v = 7.5$ m/s

at $x = 0.45$ m $\rightarrow v = 15$ m/s

Find : ① Value of const. k

② $v(x)$

Soln.

Note $a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \cdot \frac{dv}{dx}$

$\therefore a = v\frac{dv}{dx} = \frac{dv}{dt}$

$\therefore a = 600x + 600kx^3$

$\int v\,dv = \int (600x + 600kx^3)\,dx$

$v^2 = 300x^2 - 150kx^4 + C$

$\therefore$ at $v = 7.5$ m/s $\rightarrow x = 0$

$(7.5)^2 = C \Rightarrow \boxed{C = 56.25}$

at $x = 0.45$, $v = 15$

$$(15)^2 = 300(0.45) - 150(0.45)k + 56.25$$

$$150(0.45)k = -33.75$$

$$\boxed{k = -1/2}$$

$$\therefore v^2 = 300x^2 + 75x^3 + 56.25$$

$$v = \sqrt{300x^2 + 75x^3 + 56.25}$$

Prob. (11.25) P.615 : A particle moves in a St. line $a = -k\sqrt{v}$

at $t=0 \rightarrow x_0 = 0$, $v_0 = 25$ m/s

$x = 6$ m, $v_{x=6} = 12$ m/s

calculate: (1) velocity at $x = 8$ m

(2) Time required to come to rest ($v = 0$)

Soln

$$a = -k v^{1/2}$$

$$\int \frac{dv}{v^{1/2}} = \int -k \, dt$$

$$2v^{1/2} = -kt + C$$

$\therefore$ at $t=0 \rightarrow v_0 = 25 \Rightarrow \boxed{C = 0}$

$$\boxed{2v^{1/2} = -kt}$$

$$\int v^{1/2} dv = \int -k \, dx$$

$$\frac{2}{3}v^{3/2} = -kx + C$$

$$2/3 (25)^{3/2} = C \Rightarrow \boxed{C = \frac{250}{3}}$$

at $x=6$, $v_{x=6} = 12$

$$\therefore k = 9.27 \Rightarrow \frac{2}{3}v^{3/2} = -9.27x + \frac{250}{3}$$

at $x = 8$ $\therefore \frac{2}{3}v^{3/2} = (-9.27)(8) + \frac{250}{3}$

$$v = 5.7422 \text{ m/s}$$

To calculate Relation between Velocity & Time

$$a = \frac{dv}{dt} = -k\sqrt{v} = -9.75\, v^{1/2}$$

$$\int_{25}^{v} \frac{dv}{v^{1/2}} = \int_{0}^{t} -9.27\, dt$$

$$2\left|\sqrt{v}\right|_{25}^{v} = -9.27\left|t\right|_{0}^{t}$$

$$2(\sqrt{v} - 5) = -9.27\, t$$

$\therefore$ at $v=0$ $\quad -10 = -9.27\, t$

$$t = 1.0787 \text{ sec.}$$

Another Ans for (11.25)

$$a = -k\sqrt{v}$$

at $t=0 \rightarrow x_0 = 0 \rightarrow v_0 = 25$ m/s

$x = 6 \rightarrow v_{x=6} = 12$ m/s

$$\therefore a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} = -k\sqrt{v}$$

$$\int_{12}^{v} \frac{v}{v^{1/2}}\, dv = -k\int_{6}^{x} dx$$

$$= \frac{2}{3}\left|v^{3/2}\right|_{12}^{v} = -k\left|x\right|_{6}^{x} \Longrightarrow \frac{2}{3}(v^{3/2} - 41.569) = -k(x-6)$$

$\therefore$ at $x=0 \quad v_0 = 25$

$$\frac{2}{3}(25^{3/2} - 41.569) = -k(-6)$$

$$55.62 = 6K \Rightarrow \boxed{K = 9.27}$$

$\therefore$ at $x=8$

$$\frac{2}{3}(v^{3/2} - 41.569) = -9.27(8-6) \Rightarrow v = 5.7419 \text{ m/s}$$

$$a = -9.27\sqrt{v}$$

$$\frac{dv}{dt} = -9.27\sqrt{v}$$

$$\int_{25}^{v} \frac{dv}{\sqrt{v}} = -9.27 \int_0^t dt$$

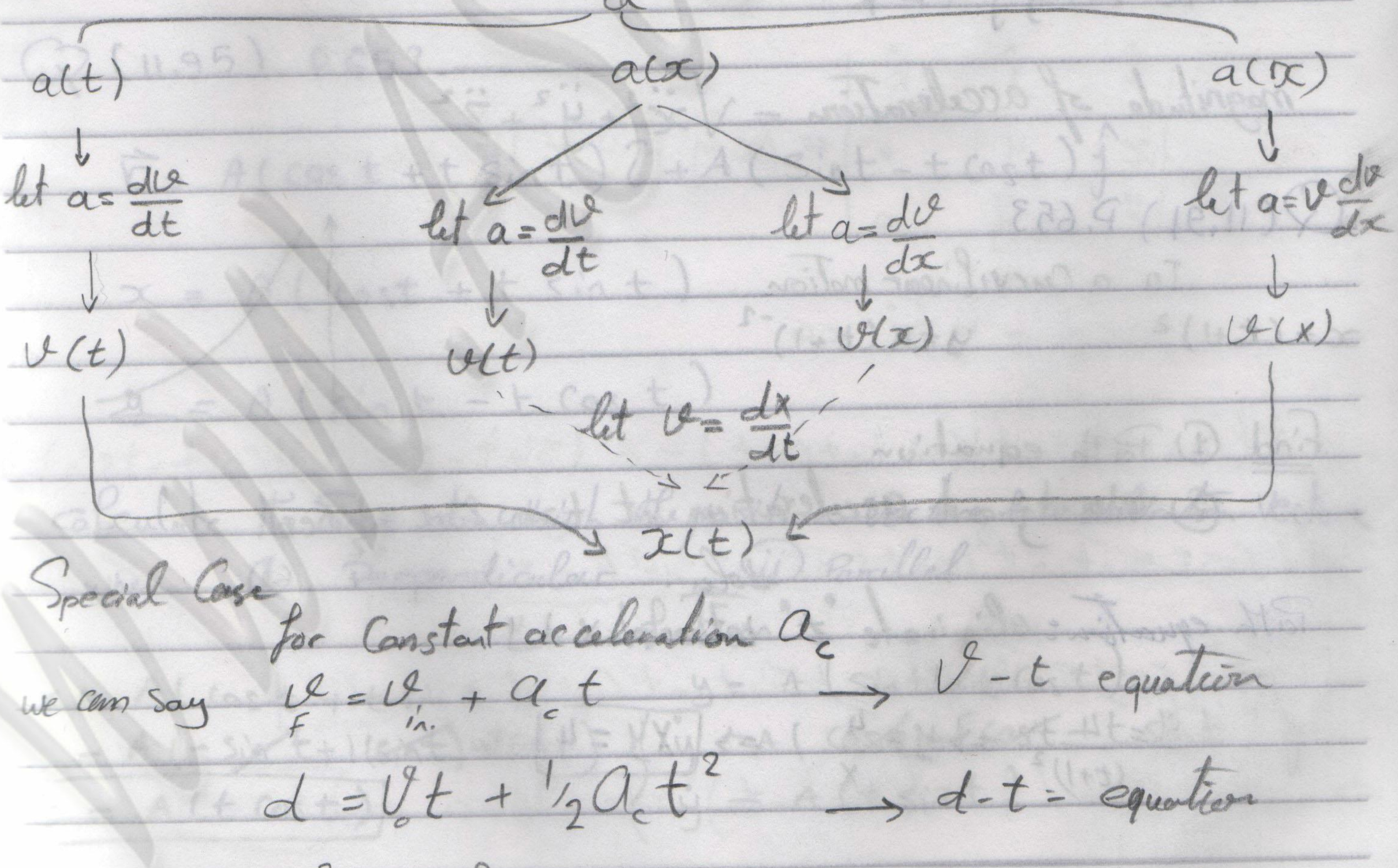
$$2\left|v^{1/2}\right|_{25}^{v} = -9.27\left|t\right|_0^t$$

$$2(v^{1/2} - 5) = -9.27t \xRightarrow{at\ v=0} -10 = -9.27t \Rightarrow t = 1.0787$$

$a = 0$, $v = 0$ ⟸ Cont. stop

$a = 0$, $v = \pm Ve$ ← Stantaneous stop

$a = \pm Ve$, $v = 0$ ← Stantaneous stop



Special Case

for Constant accaleration a_c

we can say $v_F = v_{in.} + a_c t$ → v - t equation

$d = v_0 t + \frac{1}{2} a_c t^2$ → d - t = equation

$v_f^2 = v_i^2 + 2 a_c d$ → v - d equatio

$$(15)^2 = 300(0.45) - 150(0.45)k + 56.25$$

$$150(0.45)k = -33.75$$

$$\boxed{k = -1/2}$$

$$\therefore v^2 = 300x^2 + 75x^3 + 56.25$$

$$v = \sqrt{300x^2 + 75x^3 + 56.25}$$

Prob. (11.25) P.615 : A particle moves in a st. line $a = -k\sqrt{v}$

at $t=0 \rightarrow x_0 = 0$, $v_0 = 25$ m/s

$x = 6$ m, $v_{x=6} = 12$ m/s

calculate: (1) velocity at $x = 8$ m

(2) Time required to come to rest ($v = 0$)

soln

$$a = -k v^{1/2}$$

$$\int \frac{dv}{v^{1/2}} = \int -k \, dt$$

$$2v^{1/2} = -kt + C$$

$\therefore$ at $t=0 \rightarrow v_0 = 25 \Longrightarrow \boxed{C = 0}$

$$\boxed{2v^{1/2} = -kt}$$

$$\int v^{1/2} dv = \int -k \, dx$$

$$\frac{2}{3}v^{3/2} = -kx + C$$

$$2/3 (25)^{3/2} = C \Rightarrow \boxed{C = \frac{250}{3}}$$

at $x=6$, $v_{x=6} = 12$

$$\therefore k = 9.27 \Longrightarrow \frac{2}{3}v^{3/2} = -9.27x + \frac{250}{3}$$

at $x=8$ $\quad \therefore \frac{2}{3}v^{3/2} = (-9.27)(8) + \frac{250}{3}$

$$v = 5.7422 \text{ m/s}$$